

# The stability of thin-shell wormholes with a phantom-like equation of state

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This paper discusses the stability to linearized radial perturbations of spherically symmetric thin-shell wormholes with a “phantom-like” equation of state for the exotic matter at the throat:  $\mathcal{P} = \omega\sigma$ ,  $\omega < 0$ , where  $\sigma$  is the energy-density of the shell and  $\mathcal{P}$  the lateral pressure. This equation is analogous to the generalized Chaplygin-gas equation of state used by E.F. Eiroa. The analysis, which differs from Eiroa’s in its basic approach, is carried out for wormholes constructed from the following spacetimes: Schwarzschild, de Sitter and anti de Sitter, Reissner-Nordström, and regular charged black-hole spacetimes, followed by black holes in dilaton and generalized dilaton-axion gravity.

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## I. INTRODUCTION

A powerful theoretical method for describing or mathematically constructing a class of spherically symmetric wormholes from black-hole spacetimes was proposed by Visser in 1989 [1]. This type of wormhole, constructed by the so-called cut-and-paste technique, is commonly known as a *thin-shell wormhole*, since the construction calls for grafting two black-hole spacetimes together. The junction surface is a three-dimensional thin shell. The cut-and-paste technique is now considered standard.

While there had already been a number of forerunners, the concept of a traversable wormhole was proposed by Morris and Thorne in 1988 [2]. Ten years later a renewed interest was sparked by the discovery that our Universe is undergoing an accelerated expansion [3, 4]:  $\ddot{a} > 0$  in the Friedmann equation  $\ddot{a}/a = -\frac{4\pi}{3}(\rho + 3p)$ . (Our units are taken to be those in which  $G = c = 1$ .) The acceleration is caused by a negative pressure *dark energy* with equation of state (EoS)  $p = \omega\rho$ ,  $\omega < -\frac{1}{3}$ , and  $\rho > 0$ . A value of  $\omega < -\frac{1}{3}$  is required for an accelerated expansion, while  $\omega = -1$  corresponds to a cosmological constant [5]. The case  $\omega < -1$  is referred to as *phantom energy* and leads to a violation of the null energy condition, a primary prerequisite for the existence of wormholes. Wormholes may also be supported by a generalized Chaplygin gas [6] whose EoS is  $p = -A/\rho^\alpha$ , where  $A > 0$  and  $0 < \alpha \leq 1$ .

In a thin-shell wormhole the exotic matter is confined to the thin shell. This suggests assigning an equation of state to the exotic matter on the shell. Eiroa [7] used the above generalized Chaplygin EoS  $\mathcal{P} = A/|\sigma|^\alpha$ , where  $\sigma$  is the (negative) energy-density of the shell and  $\mathcal{P}$  the lateral pressure, to perform a stability analysis for linearized radial perturbations. In this paper we will consider, analogously, the EoS  $\mathcal{P} = \omega\sigma$ ,  $\omega < 0$ , which will be called a *phantom-like* equation of state. The stabil-

ity analysis will be carried out for several spacetimes: Schwarzschild, de Sitter and anti de Sitter, Reissner-Nordström, and regular charged black hole spacetimes, as well as black holes in dilaton and generalized dilaton-axion gravity. The phantom-like equation of state yields explicit closed-form expressions for  $\sigma$ . Our approach to the stability analysis is therefore different from Eiroa’s.

## II. THIN-SHELL WORMHOLE CONSTRUCTION

Our starting point is the spherically symmetric metric [7]

$$ds^2 = -f(r)dt^2 + [f(r)]^{-1}dr^2 + h(r)(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where  $f(r)$  and  $h(r)$  are positive functions of  $r$  and  $h(r)$  is increasing. (In Sections III-VI,  $h(r) = r^2$ .) As in Ref. [8], the construction begins with two copies of a black-hole spacetime and removing from each the four-dimensional region

$$\Omega^\pm = \{r \leq a \mid a > r_h\}, \quad (2)$$

where  $r = r_h$  is the (outer) event horizon of the black hole. Now identify (in the sense of topology) the time-like hypersurfaces

$$\partial\Omega^\pm = \{r = a \mid a > r_h\}.$$

The resulting manifold is geodesically complete and possesses two asymptotically flat regions connected by a throat. Next, we use the Lanczos equations [1, 7–15]

$$S^i_j = -\frac{1}{8\pi} ([K^i_j] - \delta^i_j [K]), \quad (3)$$

where  $[K_{ij}] = K_{ij}^+ - K_{ij}^-$  and  $[K]$  is the trace of  $K^i_j$ . In terms of the surface energy-density  $\sigma$  and the surface pressure  $\mathcal{P}$ ,  $S^i_j = \text{diag}(-\sigma, \mathcal{P}, \mathcal{P})$ . The Lanczos equations now yield

$$\sigma = -\frac{1}{4\pi} [K^\theta_\theta] \quad (4)$$

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and

$$\mathcal{P} = \frac{1}{8\pi} ([K^\tau_\tau] + [K^\theta_\theta]). \quad (5)$$

A dynamic analysis can be obtained by letting the radius  $r = a$  be a function of time [8]. As a result,

$$\sigma = -\frac{1}{2\pi a} \sqrt{f(a) + \dot{a}^2} \quad (6)$$

and

$$\mathcal{P} = -\frac{1}{2}\sigma + \frac{1}{8\pi} \frac{2\ddot{a} + f'(a)}{\sqrt{f(a) + \dot{a}^2}}. \quad (7)$$

Since  $\sigma$  is negative on the shell, we are dealing with exotic matter. In fact, the weak energy condition (WEC) is obviously violated since the radial pressure  $p$  is zero for a thin shell. (The WEC requires the stress-energy tensor  $T_{\alpha\beta}$  to obey  $T_{\alpha\beta}\mu^\alpha\mu^\beta \geq 0$  for all time-like vectors and, by continuity, all null vectors.) So for the radial outgoing null vector  $(1, 1, 0, 0)$ , we therefore have  $T_{\alpha\beta}\mu^\alpha\mu^\beta = \rho + p = \sigma + 0 < 0$ .

### III. SCHWARZSCHILD WORMHOLES

For our first case, the Schwarzschild spacetime,  $h(r) = r^2$  in line element (1), as noted earlier. Also, recall that the radius  $r = a$  is a function of time. It is easy to check that  $\mathcal{P}$  and  $\sigma$  obey the conservation equation

$$\frac{d}{d\tau}(\sigma a^2) + \mathcal{P} \frac{d}{d\tau}(a^2) = 0.$$

(In Eqs. (6) and (7), the overdot denotes the derivative with respect to  $\tau$ .) The equation can be written in the form

$$\frac{d\sigma}{da} + \frac{2}{a}(\sigma + \mathcal{P}) = 0. \quad (8)$$

For a static configuration of radius  $a_0$ , we have  $\dot{a} = 0$  and  $\ddot{a} = 0$ . Moreover, we will consider linearized fluctuations around a static solution characterized by the constants  $a_0$ ,  $\sigma_0$ , and  $\mathcal{P}_0$ . Given the EoS  $\mathcal{P} = \omega\sigma$ , Eq. (8) can be solved by separation of variables to yield

$$|\sigma(a)| = |\sigma_0| \left(\frac{a_0}{a}\right)^{2(\omega+1)},$$

where  $\sigma_0 = \sigma(a_0)$ . So the solution is

$$\sigma(a) = \sigma_0 \left(\frac{a_0}{a}\right)^{2(\omega+1)}, \quad \sigma_0 = \sigma(a_0). \quad (9)$$

Next, we rearrange Eq. (6) to obtain the equation of motion

$$\dot{a}^2 + V(a) = 0.$$

Here the potential  $V(a)$  is defined as

$$V(a) = f(a) - [2\pi a\sigma(a)]^2. \quad (10)$$

Expanding  $V(a)$  around  $a_0$ , we obtain

$$V(a) = V(a_0) + V'(a_0)(a - a_0) + \frac{1}{2}V''(a_0)(a - a_0)^2 + O[(a - a_0)^3]. \quad (11)$$

Since we are linearizing around  $a = a_0$ , we require that  $V(a_0) = 0$  and  $V'(a_0) = 0$ . The configuration is in stable equilibrium if  $V''(a_0) > 0$ .

Now recall that for the Schwarzschild spacetime,  $f(r) = 1 - 2M/r$ . It follows that

$$\begin{aligned} V(a) &= 1 - \frac{2M}{a} - 4\pi^2 a^2 \sigma^2 \\ &= 1 - \frac{2M}{a} - 4\pi^2 a^2 \sigma_0^2 \left(\frac{a_0}{a}\right)^{4+4\omega} \end{aligned}$$

from Eq. (9). From Eq. (6) with  $\dot{a} = 0$ ,

$$\sigma_0 = -\frac{1}{2\pi a_0} \sqrt{1 - \frac{2M}{a_0}},$$

so that

$$V(a) = 1 - \frac{2M}{a} - \left(1 - \frac{2M}{a_0}\right) \frac{a_0^{2+4\omega}}{a^{2+4\omega}}. \quad (12)$$

The first requirement,  $V(a_0) = 0$ , is clearly met, but not the second. (If the exotic matter on the shell were not required to meet the extra condition in the form of an EoS, then  $V'(a_0)$  would indeed be zero [8].) From

$$\begin{aligned} V'(a) &= \\ \frac{2M}{a_0^2} - \left(1 - \frac{2M}{a_0}\right) a_0^{2+4\omega} (-2 - 4\omega) a_0^{-3-4\omega} &= 0, \end{aligned}$$

we obtain the condition

$$\omega = -\frac{1}{2} \frac{a_0/M - 1}{a_0/M - 2}. \quad (13)$$

Observe that as  $a_0 \rightarrow +\infty$ ,  $\omega \rightarrow -1/2-$ , and as  $a_0 \rightarrow 2M+$ ,  $\omega \rightarrow -\infty$ . At  $a_0 = 3M$ ,  $\omega = -1$ . Substituting in

$$\begin{aligned} V''(a) &= \\ -\frac{4M}{a^3} - \left(1 - \frac{2M}{a_0}\right) a_0^{2+4\omega} (2 + 4\omega)(3 + 4\omega) a^{-4-4\omega} \end{aligned}$$

and simplifying, we obtain the intermediate result

$$V''(a_0) = \frac{2}{a_0^2} \left( -\frac{2}{a_0/M} + \frac{1}{a_0/M} \frac{a_0/M - 4}{a_0/M - 2} \right) > 0. \quad (14)$$

Since the Schwarzschild black hole has an event horizon at  $r = 2M$ ,  $a_0/M - 2 > 0$ , and we conclude that the inequality  $V''(a_0) > 0$  can only be satisfied if

$$a_0 < 0.$$

As a result, there are no stable solutions.

To allow a comparison to some of the other cases, let us choose (arbitrarily)  $a_0/M = 5$ , as a result of which  $\omega = -2/3$ , and plot  $V(a)$  against  $a/M$ , as shown in Fig. 1.

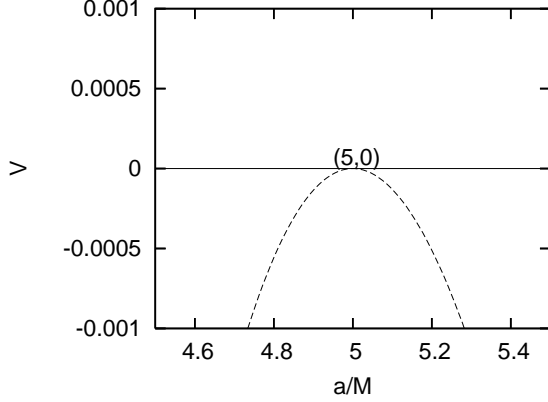


FIG. 1: The wormhole is unstable.

The more general analysis in Ref. [8] depends on the parameter  $\beta^2(\sigma) = \partial\mathcal{P}/\partial\sigma$ , where  $\beta$  is usually interpreted as the speed of sound, so that  $0 < \beta^2 \leq 1$ . There are no stable solutions in this range. However, as discussed in Ref. [8], since we are dealing with exotic matter, this assumption may be questioned, that is,  $\beta^2$  may be just a convenient parameter. In that case, some stable configurations may not be out of question. Our additional assumption, the EoS  $\mathcal{P} = \omega\sigma$  on the shell, eliminates this possibility.

#### IV. WORMHOLES WITH A COSMOLOGICAL CONSTANT

##### A. Schwarzschild-de Sitter spacetimes

In the presence of a cosmological constant,  $f(r) = 1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2$ . For the de Sitter case,  $\Lambda > 0$ . To keep  $f(r)$  from becoming negative, we must have  $\Lambda M^2 \leq 1/9$ . This condition results in two event horizons, where the inner horizon is between  $2M$  and  $3M$ . (See Ref. [7] for details.) We therefore assume that  $a$  is greater than the outer horizon. Proceeding as in Sec. III,

$$V(a) = 1 - \frac{2M}{a} - \frac{1}{3}\Lambda a^2 - \left(1 - \frac{2M}{a_0} - \frac{1}{3}\Lambda a_0^2\right) \left(\frac{a_0}{a}\right)^{2+4\omega}. \quad (15)$$

Observe that  $V(a_0) = 0$ . As before, we have to determine the condition on  $\omega$  so that  $V'(a_0) = 0$ :

$$\omega = -\frac{1}{2} \frac{1 - 1/(a_0/M) - (2/3)\Lambda M^2(a_0/M)^2}{1 - 2/(a_0/M) - (1/3)\Lambda M^2(a_0/M)^2}. \quad (16)$$

(As in the Schwarzschild case, as  $a_0 \rightarrow +\infty$ ,  $\omega \rightarrow -1/2$ -, and  $\omega \rightarrow -\infty$  as  $a_0$  approaches the outer event horizon.) Substituting in  $V''(a_0)$  and simplifying, we get

$$V''(a_0) = \frac{2}{a_0^2} \frac{-1/(a_0/M) + 3\Lambda M^2(a_0/M) - (2/3)\Lambda M^2(a_0/M)^2}{1 - 2/(a_0/M) - (1/3)\Lambda M^2(a_0/M)^2} > 0. \quad (17)$$

The form of  $V''(a_0)$  forces us to consider two cases, a positive and negative denominator.

If the denominator is positive, then

$$\Lambda M^2 > \frac{1}{(a_0/M)[3(a_0/M) - (2/3)(a_0/M)^2]}. \quad (18)$$

This inequality implies that  $a_0/M < 4.5$  to keep the right side positive. It is easy to show analytically that  $\Lambda M^2 > 1/9$ ; in fact,  $(3, 1/9)$  is a minimum. We may also plot  $\Lambda M^2$  against  $a/M$ , as shown in Fig. 2. So for this case,

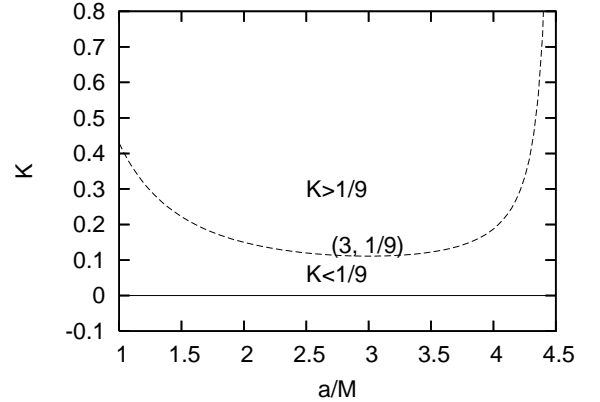


FIG. 2:  $K = \Lambda M^2$  is plotted against  $a/M$ .

the condition  $V''(a_0) > 0$  cannot be met (since we must have  $\Lambda M^2 \leq 1/9$ ), and we get only unstable solutions. Plotting  $V(a)$  around  $a_0/M = 5$  yields a graph that is very similar to the graph in Fig. 1.

For the second case,

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2 \left(\frac{a_0}{M}\right)^2 < 0 \quad (19)$$

in inequality (17) we obtain

$$\Lambda M^2 \left[3 \left(\frac{a_0}{M}\right) - \frac{2}{3} \left(\frac{a_0}{M}\right)^2\right] < \frac{1}{a_0/M}. \quad (20)$$

If  $a_0/M > 4.5$ , then the left side is negative, and the condition is automatically satisfied. If  $a_0/M < 4.5$ , then, according to Fig. 2,  $\Lambda M^2 < 1/9$ , the region below the graph. So we conclude that in the second case, the wormholes are stable.

For comparison, let us choose  $a_0/M = 5$  again and  $\Lambda M^2 = 0.11 < 1/9$ , resulting in  $\omega = -1.63$ . The plot of  $V(a)$  against  $a/M$  is shown in Fig. 3.

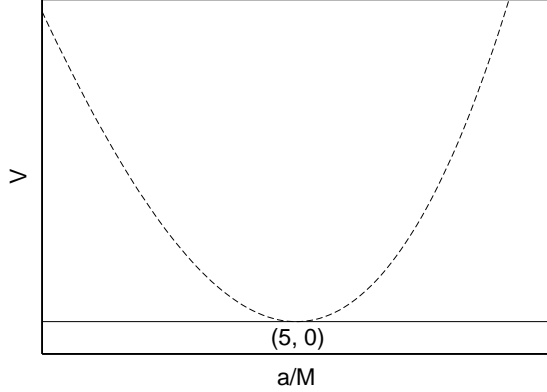


FIG. 3: The wormhole is stable.

In summary, in the Schwarzschild-de Sitter case, the thin-shell wormholes are stable if, and only if,

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2 \left(\frac{a_0}{M}\right)^2 < 0.$$

### B. Schwarzschild-anti de Sitter spacetimes

To study the case  $\Lambda < 0$ , we return to inequality (17) and consider first a negative denominator:

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2 \left(\frac{a_0}{M}\right)^2 < 0.$$

Solving for  $\Lambda M^2$ , we obtain

$$\Lambda M^2 > \frac{3 - 6/(a_0/M)}{(a_0/M)^2}.$$

Since  $a_0/M > 2$ , we conclude that  $\Lambda M^2 > 0$ , so that this case cannot arise.

Reversing the sense of the inequality, we have from inequality (17)

$$\Lambda M^2 \left[ 3 \left(\frac{a_0}{M}\right) - \frac{2}{3} \left(\frac{a_0}{M}\right)^2 \right] > \frac{1}{a_0/M}.$$

Then the second factor on the left must be negative, which implies that  $a_0/M > 4.5$ .

So the wormhole is stable whenever

$$(1) \quad \Lambda M^2 < \frac{1}{(a_0/M)[3(a_0/M) - (2/3)(a_0/M)^2]}$$

and

$$(2) \quad a_0/M > 4.5.$$

## V. REISSNER-NORDSTRÖM WORMHOLES

If the starting point is a Reissner-Nordström spacetime, then

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \quad (21)$$

where  $M$  and  $Q$  are the mass and charge, respectively, of the black hole. For  $0 < |Q| < M$ , this black hole has two event horizons at  $r = M \pm \sqrt{M^2 - Q^2}$ . As usual, we require that  $r = a$  is larger than the outer horizon.

Here we have

$$V(a) = 1 - \frac{2M}{a} + \frac{Q^2}{a^2} - \left(1 - \frac{2M}{a_0} + \frac{Q^2}{a_0^2}\right) \left(\frac{a_0}{a}\right)^{2+4\omega}. \quad (22)$$

Once again,  $V(a_0) = 0$ . From  $V'(a_0) = 0$  we obtain

$$\omega = -\frac{1}{2} \frac{(a_0/M)^2 - a_0/M}{(a_0/M)^2 - 2(a_0/M) + Q^2/M^2}. \quad (23)$$

Substituting into  $V''(a_0)$  and simplifying, yields the following inequality:

$$V''(a_0) = \frac{2}{a_0^2} \frac{-a_0/M - (Q^2/M^2)[1/(a_0/M)] + 2Q^2/M^2}{(a_0/M)^2 - 2(a_0/M) + Q^2/M^2} > 0. \quad (24)$$

Since  $a_0/M > 2$ , the denominator is positive. Solving for  $Q^2/M^2$ , leads to

$$\frac{|Q|}{M} > \frac{a_0/M}{\sqrt{2(a_0/M) - 1}}, \quad (25)$$

which exceeds unity. To meet this condition,  $|Q|$  would have to exceed  $M$ .

So to obtain a stable solution, we will have to tolerate a naked singularity at  $r = 0$ , but since  $a_0 > 0$ , the naked singularity is removed from the wormhole spacetime.

## VI. WORMHOLES FROM REGULAR CHARGED BLACK HOLES

Thin-shell wormholes from regular charged black holes, due to Ayon-Beato and García [16], are discussed in Ref.

[17]. For this black hole,

$$f(r) = 1 - \frac{2M}{r} + \frac{2M}{r} \tanh\left(\frac{Q^2}{2Mr}\right). \quad (26)$$

Again,  $M$  and  $Q$  are the mass and charge, respectively. It is shown in Ref. [16] that the black hole has two event horizons whenever  $|Q| < 1.05M$ . Consider next

$$V(a) = 1 - \frac{2M}{a} + \frac{2M}{a} \tanh\left(\frac{Q^2}{2Ma}\right) - \left[1 - \frac{2M}{a_0} + \frac{2M}{a_0} \tanh\left(\frac{Q^2}{2Ma_0}\right)\right] \left(\frac{a_0}{a}\right)^{2+4\omega}. \quad (27)$$

As before,  $V(a_0) = 0$ , and from  $V'(a_0) = 0$ , we get

$$\omega = \frac{1}{2} \left[ -1 + \frac{g(a_0)}{a_0/M - 2 + 2 \tanh[Q^2/(2Ma_0)]} \right], \quad (28)$$

where

$$g(a_0) = -1 + \tanh\left(\frac{Q^2}{2Ma_0}\right) + \frac{Q^2/M^2}{2a_0/M} \text{sech}^2\left(\frac{Q^2}{2Ma_0}\right).$$

Based on the graphical output, we get only unstable solutions. For example, choosing  $a_0/M = 5$  again for comparison and letting  $|Q|/M = 0.9$ , we get  $\omega = -0.63$ . The resulting graph, shown in Fig. 4, resembles Fig. 1. Other choices of the parameters lead to similar results.

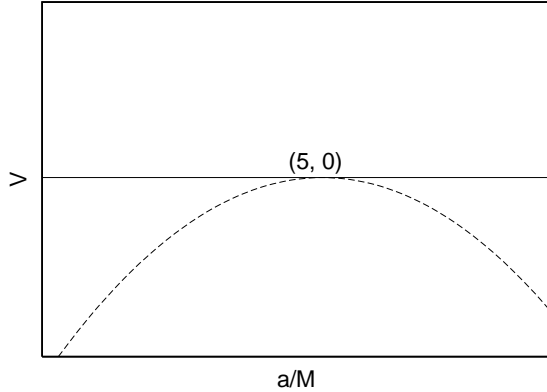


FIG. 4: The wormhole is unstable.

## VII. WORMHOLES FROM BLACK HOLES IN DILATON AND DILATON-AXION GRAVITY

Of the remaining thin-shell wormholes, based on dilaton and dilaton-axion black holes, respectively, we will

consider in detail only the latter, which is the more complicated of the two.

The dilaton-axion black-hole solution, inspired by low-energy string theory, was discovered by Sur, *et al.*, [18] and is also discussed in Ref. [19]. We need to list certain parameters in order to define  $V(a)$ . As in the Reissner-Nordström wormhole, there are two event horizons, denoted by  $r_-$  and  $r_+$ , respectively. Returning now to line element (1), we can list both  $f(r)$  and  $h(r)$  [18]:

$$f(r) = \frac{(r - r_-)(r - r_+)}{(r - r_0)^{2-2n}(r + r_0)^{2n}};$$

$$h(r) = \frac{(r + r_0)^{2n}}{(r - r_0)^{2n-2}}.$$

Since  $h(r)$  is no longer equal to  $r^2$  in line element (1), Eq. (6) becomes

$$\sigma = -\frac{1}{4\pi} \frac{h'(a)}{h(a)} \sqrt{f(a) + \dot{a}^2} \quad (29)$$

and the conservation equation (8) has to be replaced by [7]:

$$\frac{d}{d\tau}(\sigma\mathcal{A}) + \mathcal{P} \frac{d\mathcal{A}}{d\tau} = \{[h'(a)]^2 - 2h(a)h''(a)\} \frac{\dot{a}\sqrt{f(a) + \dot{a}^2}}{2h(a)}, \quad (30)$$

where  $\mathcal{A} = 4\pi h(a)$  is the area of the throat by Eq. (1). The prime and dot denote, respectively, the derivatives with respect to  $a$  and  $\tau$ . Substituting Eq. (29) on the right-hand side, we get

$$\begin{aligned} \frac{d}{d\tau}[4\pi h(a)\sigma] + \mathcal{P} \frac{d}{d\tau}[4\pi h(a)] \\ = -\{[h'(a)]^2 - 2h(a)h''(a)\} \frac{\dot{a}(4\pi\sigma)}{2h'(a)}, \end{aligned}$$

whence

$$\begin{aligned} \frac{d}{da}[\sigma h(a)] + \mathcal{P} \frac{d}{da}[h(a)] \\ = -\{[h'(a)]^2 - 2h(a)h''(a)\} \frac{\sigma}{2h'(a)}. \end{aligned}$$

Our final form is

$$\begin{aligned} h(a)\sigma' + h'(a)(\sigma + \mathcal{P}) \\ + \{[h'(a)]^2 - 2h(a)h''(a)\} \frac{\sigma}{2h'(a)}. \end{aligned} \quad (31)$$

Making use of  $\mathcal{P} = \omega\sigma$ , this equation can be solved by separation of variables:

$$\sigma(a) = \sigma_0 \left[ \frac{h(a_0)}{h(a)} \right]^{3/2+\omega} \left[ \frac{h'(a_0)}{h'(a)} \right]^{-1}. \quad (32)$$

(Here we used the fact that  $h'(a) > 0$ .) It is shown in Ref. [17] that

$$\sigma_0 = -\frac{4[a_0 + (1 - 2n)r_0](a_0 - r_-)(a_0 - r_+)}{D(a_0 - r_0)(a_0 + r_0)}, \quad (33)$$

where

$$D = 8\pi(a_0 - r_0)^{1-n}(a_0 + r_0)^n \sqrt{(a_0 - r_-)(a_0 - r_+)}. \quad (34)$$

Using the equation of motion  $\dot{a}^2 + V(a) = 0$  once again, we get from Eq. (29),

$$V(a) = f(a) - \left[ 4\pi \frac{h(a)}{h'(a)} \sigma(a) \right]^2. \quad (35)$$

Eq. (32) now yields

$$V(a) = \frac{(a - r_-)(a - r_+)}{(a - r_0)^{2-2n}(a + r_0)^{2n}} - \left[ 4\pi \frac{h(a)}{h'(a)} \sigma_0 \right]^2 \left[ \frac{h(a_0)}{h'(a)} \right]^{3+2\omega}. \quad (36)$$

While it is easy enough to check that  $V(a_0) = 0$ , it is no longer convenient to compute  $\omega$  as a function of the various parameters. Plotting  $V(a)$  against  $a$  instead of  $a/M$ , we can determine  $\omega$  by trial and error:  $V(a)$  must be tangent to the  $a$ -axis at  $a = a_0$ , where  $V(a_0) = 0$  automatically. For example, if  $a_0 = 5$ ,  $r_0 = 1$ ,  $r_- = 2$ ,  $r_+ = 2.05$ , and  $n = 0.8$ , then  $\omega = -0.915$ . If  $a_0 = 5$ ,  $r_0 = 1$ ,  $r_- = 2$ ,  $r_+ = 3$ , and  $n = 0.8$ , then  $\omega = -1.132$ . Reducing  $n$  to 0.6 produces  $\omega = -0.84$  in the first case and  $\omega = -1.041$  in the second. In all cases the graphs are concave down at  $a_0 = 5$  and look similar to the graph in Fig. 1. Based on the graphical output, there do not appear to be any stable solutions.

For the dilaton case we have [20]

$$V(a) = \left( 1 - \frac{A}{a} \right) \left( 1 - \frac{B}{a} \right)^{(1-b^2)/(1+b^2)} - \left[ 4\pi \frac{h(a)}{h'(a)} \sigma_0 \right]^2 \left[ \frac{h(a_0)}{h'(a)} \right]^{3+2\omega}, \quad (37)$$

where  $h(a) = a^2(1 - B/a)^{2b^2/(1+b^2)}$  for various constants. Once again, one can readily check that  $V(a_0) = 0$ .

As in the dilaton-axion case,  $\omega$  can be found by trial and error. For example, if  $a_0 = 5$ ,  $b = 0.5$ ,  $A = 2$ , and  $B = 1$ , then  $\omega = -0.693$ ; if  $a_0 = 6$ ,  $b = 0.8$ ,  $A = 4$ , and  $B = 2$ , then  $\omega = -0.94$ , etc. The resulting graphs are similar to those in the dilaton-axion case.

## VIII. CONCLUSION

This paper discusses the stability to linearized radial perturbations of spherically symmetric thin-shell wormholes with the equation of state  $\mathcal{P} = \omega\sigma$ ,  $\omega < 0$ , for the exotic matter at the throat. This EoS is referred to as phantom-like. Various spacetimes were considered.

It was found that the wormholes are unstable if constructed from Schwarzschild spacetimes, as well as from black holes in dilaton and dilaton-axion gravity. For the Reissner-Nordström case, stable solutions exist only if

$$\frac{|Q|}{M} > \frac{a_0/M}{\sqrt{2(a_0/M) - 1}},$$

leading to a naked singularity. For the Schwarzschild-de Sitter case, the wormholes are stable if, and only if,

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2 \left( \frac{a_0}{M} \right)^2 < 0.$$

In the Schwarzschild-anti de Sitter case, the configurations are stable whenever

$$(1) \quad \Lambda M^2 < \frac{1}{(a_0/M)[3(a_0/M) - (2/3)(a_0/M)^2]}$$

and

$$(2) \quad a_0/M > 4.5.$$

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- [1] M. Visser, Nucl. Phys. B **328**, 203 (1989).
  - [2] M.S. Morris and K.S. Thorne, Amer. J. Phys. **56**, 395 (1988).
  - [3] A.G. Riess, A.V. Filippenko, P. Challis, *et al.*, Astron. J. **116**, 1009 (1998).
  - [4] S.J. Perlmutter, G. Aldering, G. Goldhaber, *et al.*, Astrophys. J. **517**, 565 (1999).
  - [5] M. Carmeli, arXiv: astro-ph/0111259.
  - [6] F.S.N. Lobo, Phys. Rev. D **73**, 064028 (2006).
  - [7] E.F. Eiroa, Phys. Rev. D **80**, 044033 (2009).
  - [8] E. Poisson and M. Visser, Phys. Rev. D **52**, 7318 (1995).
  - [9] F.S.N. Lobo and P. Crawford, Class. Quant. Grav. **21**, 391 (2004).
  - [10] E.F. Eiroa and G.E. Romero, Gen. Rel. Grav. **36**, 651 (2004).
  - [11] M. Thibeault, C. Simeone, and E.F. Eiroa, Gen. Rel. Grav. **38**, 1593 (2006).
  - [12] F. Rahaman, M. Kalam, and S. Chakraborty, Gen. Rel. Grav. **38**, 1687 (2006).
  - [13] F. Rahaman, M. Kalam, and S. Chakraborty, Chin. J. Phys. **45**, 518 (2007).
  - [14] M. G. Richarte and C. Simeone, Phys. Rev. D **76**, 087502 (2007).
  - [15] J. P. S. Lemos and F.S.N. Lobo, Phys. Rev D **78**, 044030

- (2008).
- [16] E. Ayon-Beato and A. García, Phys. Lett. B **464**, 25 (1999).
- [17] F. Rahaman, K.A. Rahman, Sk. A. Rakib, and Peter K.F. Kuhfittig, arXiv: 0909.1071.
- [18] S. Sur, S. Das, and S. SenGupta, JHEP **10**, 064 (2005).
- [19] A.A. Usmani, Z. Hasan, F. Rahaman, Sk. A. Rakib, and Peter K.F. Kuhfittig, Gen. Rel. Grav. (accepted), arXiv: 1001.1415.
- [20] E.F. Eiroa, Phys. Rev. D **78**, 024018 (2008).